2024-2025
Fall Semester

Course of Power System Analysis

Study of asymmetrical three-phase circuits: the sequence decomposition

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Outline

Transformation of a three-phase system with symmetrical components

Considerations for Grounding

Decoupling of an asymmetrical three-phase impedance system

Apparent power invariance property

Transformation of a three-phase system with symmetrical components

The study of a three-phase system is not so difficult **if the voltages and currents constitute a balanced and symmetrical system**. In this case, the study of a three-phase system can be reduced to the study of **one phase.**

Recall:

Definition: **balanced system** – in a balanced system, the sum of the three phasors of currents or voltages is zero.

Definition: **symmetrical system** – in a symmetrical system, the angles between subsequent phasors of voltages or currents are equal.

Transformation of a three-phase system with symmetrical components

The study of a three-phase system is not so difficult **if the voltages and currents constitute a balanced and symmetrical system**. In this case, the study of a three-phase system can be reduced to the study of **one phase.**

If the three-phase system under study is **not balanced and/or asymmetrical**, then **all three coupled phases** must be studied. This involves solving systems of equations with complex variables. The transformation with symmetrical components **reduces the computational difficulty in the case of an asymmetrical and/or unbalanced system → decoupling the equations.**

Transformation of a three-phase system with symmetrical components

Symmetrical components allow for **simplification of three-phase system study**, and, in general, with ***n* phases**, which are **asymmetrical** and **unbalanced** in voltages and currents. In the following, we consider *only three-phase systems.*

Definition: **three-phase system** - Triplet of complex numbers representing an electrical system (i.e., simple or compound voltages, phase currents, impedances).

We consider three unequal generic complex numbers $\bar{V}_a, \bar{V}_b, \bar{V}_c$ Î » representing three quantities in a three-phase system.

Transformation of a three-phase system with symmetrical components

We define the vector $\left[\bar{V}_{abc}\right] \in$ »^{3}, where »^{3} represents a three-dimensional vector space bounded by the complex number field » .

$$\left[\bar{V}_{abc}\right] = \begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix}$$

From now on, **references to a three-phase system and the preceding vector are equivalent.**
We will show how to break down a three-phase system into its three symmetrical, balanced three-phase systems, called ***sequences***.

Transformation of a three-phase system with symmetrical components

Hypotheses:

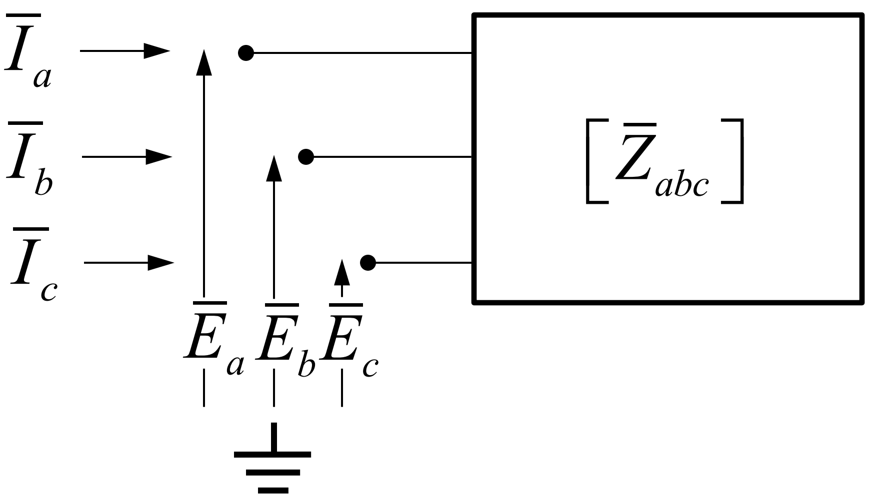
- Generic sinusoidal network in steady-state with **unbalanced and asymmetrical voltages and currents**;
- Network with **linear components**;
- **Common ground for all voltage levels** (there is no system with isolated neutral).

In general, each network element can be modeled by its **impedance matrix:**

$$\begin{bmatrix} \bar{E}_a \\ \bar{E}_b \\ \bar{E}_c \end{bmatrix} = \begin{bmatrix} \bar{Z}_{aa} & \bar{Z}_{ab} & \bar{Z}_{ac} \\ \bar{Z}_{ba} & \bar{Z}_{bb} & \bar{Z}_{bc} \\ \bar{Z}_{ca} & \bar{Z}_{cb} & \bar{Z}_{cc} \end{bmatrix} \begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix}$$

Transformation of a three-phase system with symmetrical components

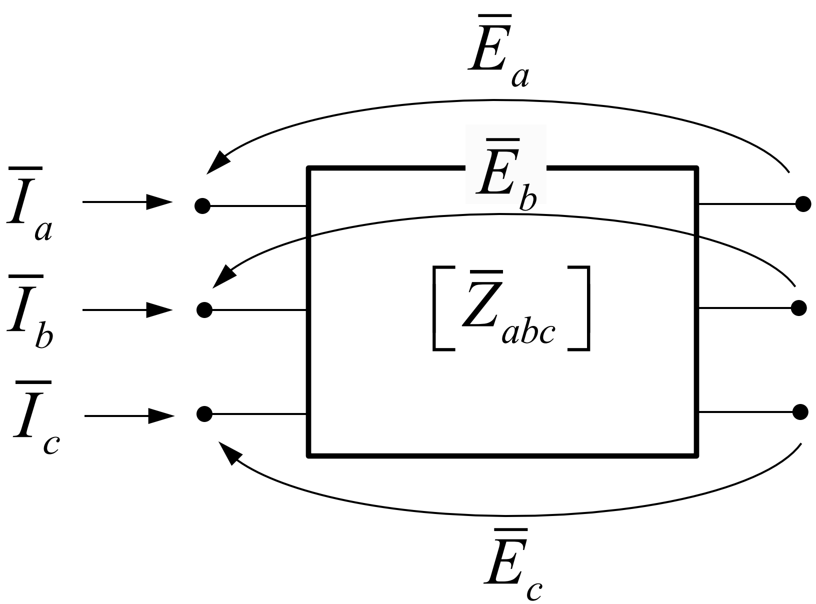
In the case of **single-port** components (i.e. motors, generators, loads, etc.), the voltages $\bar{E}_a, \bar{E}_b, \bar{E}_c$ are those **applied to the terminals of the component itself and to earth**, and the currents $\bar{I}_a, \bar{I}_b, \bar{I}_c$ are phase currents.



$$\begin{bmatrix} \bar{E}_a \\ \bar{E}_b \\ \bar{E}_c \end{bmatrix} = \begin{bmatrix} \bar{Z}_{aa} & \bar{Z}_{ab} & \bar{Z}_{ac} \\ \bar{Z}_{ba} & \bar{Z}_{bb} & \bar{Z}_{bc} \\ \bar{Z}_{ca} & \bar{Z}_{cb} & \bar{Z}_{cc} \end{bmatrix} \begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix}$$

Transformation of a three-phase system with symmetrical components

In the case of elements with **two ports** (i.e. lines, transformers, etc.) $\bar{E}_a, \bar{E}_b, \bar{E}_c$ are the differences between the input and output voltages, and the currents $\bar{I}_a, \bar{I}_b, \bar{I}_c$ are phase currents.



$$\begin{bmatrix} \bar{E}_a \\ \bar{E}_b \\ \bar{E}_c \end{bmatrix} = \begin{bmatrix} \bar{Z}_{aa} & \bar{Z}_{ab} & \bar{Z}_{ac} \\ \bar{Z}_{ba} & \bar{Z}_{bb} & \bar{Z}_{bc} \\ \bar{Z}_{ca} & \bar{Z}_{cb} & \bar{Z}_{cc} \end{bmatrix} \begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix}$$

Transformation of a three-phase system with symmetrical components

***Observation*:** the relationship on the previous slide implies **mutual coupling between phases**, and therefore the presence of three generic voltages leads to the flow of three generic currents. This means that **network studies cannot be carried out using single-phase equivalent circuits.**

Transformation of a three-phase system with symmetrical components

Additional assumption: network elements are, in general, made up of **impedance matrices** characterized by the following **two circulant symmetries**:

$$\bar{Z}_{aa} = \bar{Z}_{bb} = \bar{Z}_{cc} = \bar{Z}$$
$$\bar{Z}_{ab} = \bar{Z}_{ac} = \bar{Z}_{bc} = \bar{Z}_{ba} = \bar{Z}_{ca} = \bar{Z}_{cb} = \bar{M}$$

$$\left[\bar{Z}_{abc}\right] = \begin{bmatrix} \bar{Z} & \bar{M} & \bar{M} \\ \bar{M} & \bar{Z} & \bar{M} \\ \bar{M} & \bar{M} & \bar{Z} \end{bmatrix}$$

and, for rotating machines:

$$\bar{Z}_{aa} = \bar{Z}_{bb} = \bar{Z}_{cc} = \bar{Z}$$
$$\bar{Z}_{ab} = \bar{Z}_{bc} = \bar{Z}_{ca} = \bar{M}$$
$$\bar{Z}_{ac} = \bar{Z}_{ba} = \bar{Z}_{cb} = \bar{K}$$

$$\left[\bar{Z}_{abc}\right] = \begin{bmatrix} \bar{Z} & \bar{M} & \bar{K} \\ \bar{K} & \bar{Z} & \bar{M} \\ \bar{M} & \bar{K} & \bar{Z} \end{bmatrix}$$

Observation: in what follows, we derive the sequence method for the generic case of **circulant symmetric matrices** of network components, so the **latter**.

Transformation of a three-phase system with symmetrical components

The procedure for decoupling the link between voltages and currents involves calculating the **eigenvalues and eigenvectors** of the matrix $\left[\bar{Z}_{abc}\right]$

$$\begin{vmatrix} \bar{Z}-\lambda & \bar{M} & \bar{M} \\ \bar{M} & \bar{Z}-\lambda & \bar{M} \\ \bar{M} & \bar{M} & \bar{Z}-\lambda \end{vmatrix} = 0$$

Using Sarrus' method, for example, to calculate the determinant of the previous equation, we get the following **characteristic polynomial**:

$$\left(\bar{Z}-\lambda\right)^3 + 2\bar{M}^3 - 3\bar{M}^2\left(\bar{Z}-\lambda\right) = 0$$

which has a **single root** (λ_1) and another **double root** $(\lambda_{2,3})$:

$$\lambda_1 = \bar{Z} + 2\bar{M}; \lambda_2 = \lambda_3 = \bar{Z} - \bar{M}$$

Transformation of a three-phase system with symmetrical components

The **eigenvectors** of the root λ_1 are:

$$\begin{bmatrix} \bar{Z} - (\bar{Z} + 2\bar{M}) & \bar{M} & \bar{M} \\ \bar{M} & \bar{Z} - (\bar{Z} + 2\bar{M}) & \bar{M} \\ \bar{M} & \bar{M} & \bar{Z} - (\bar{Z} + 2\bar{M}) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Which is equivalent to:

$$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Necessary and sufficient condition →

$$x_1 = x_2 = x_3 = k \in \square; \quad \begin{bmatrix} k \\ k \\ k \end{bmatrix}$$

Eigenvector of the root λ_1

Transformation of a three-phase system with symmetrical components

The **eigenvectors** of the roots λ_2 and λ_3 are:

$$\begin{bmatrix} \bar{Z} - (\bar{Z} - \bar{M}) & \bar{M} & \bar{M} \\ \bar{M} & \bar{Z} - (\bar{Z} - \bar{M}) & \bar{M} \\ \bar{M} & \bar{M} & \bar{Z} - (\bar{Z} - \bar{M}) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Which is equivalent to:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Necessary and sufficient condition → $x_1 + x_2 + x_3 = 0$

Transformation of a three-phase system with symmetrical components

In the case of a circulant impedance matrix, we have:

$$\begin{vmatrix} \bar{Z}-\lambda & \bar{M} & \bar{K} \\ \bar{K} & \bar{Z}-\lambda & \bar{M} \\ \bar{M} & \bar{K} & \bar{Z}-\lambda \end{vmatrix} = 0$$

The characteristic polynomial is:

$$\left(\bar{Z}-\lambda\right)^3 + \bar{M}^3 + \bar{K}^3 - 3\bar{M}\bar{K}\left(\bar{Z}-\lambda\right) = 0$$

Transformation of a three-phase system with symmetrical components

The roots are:

$$I_1 = \bar{Z} + \bar{K} + \bar{M}$$

$$I_2 = \frac{1}{2}\left(-\sqrt{3}\underbrace{\sqrt{-K^2 + 2\bar{K}\bar{M} - M^2}}_{\sqrt{-\left(\bar{K}-\bar{M}\right)^2} = j\left(\bar{K}-\bar{M}\right)} - \bar{K} - \bar{M} + 2\bar{Z} \right) = -j\frac{\sqrt{3}}{2}\left(\bar{K}-\bar{M}\right) - \bar{K} - \bar{M} + 2\bar{Z}$$

$$I_3 = \frac{1}{2}\left(\sqrt{3}\underbrace{\sqrt{-K^2 + 2\bar{K}\bar{M} - M^2}}_{\sqrt{-\left(\bar{K}-\bar{M}\right)^2} = j\left(\bar{K}-\bar{M}\right)} - \bar{K} - \bar{M} + 2\bar{Z} \right) = j\frac{\sqrt{3}}{2}\left(\bar{K}-\bar{M}\right) - \bar{K} - \bar{M} + 2\bar{Z}$$

Transformation of a three-phase system with symmetrical components

The **eigenvectors** of the root λ_1 (**corresponding to the case of the circulant impedance matrix**) are given by:

$$\begin{bmatrix} \bar{Z} - \left(\bar{Z} + \bar{M} + \bar{K}\right) & \bar{M} & \bar{K} \\ \bar{K} & \bar{Z} - \left(\bar{Z} + \bar{M} + \bar{K}\right) & \bar{M} \\ \bar{M} & \bar{K} & \bar{Z} - \left(\bar{Z} + \bar{M} + \bar{K}\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

Sufficient condition to satisfy the above equation:

$$x_1 = x_2 = x_3 = k \in \square; \begin{bmatrix} k \\ k \\ k \end{bmatrix}$$

Transformation of a three-phase system with symmetrical components

The **eigenvectors** of the root λ_2 (**corresponding to the case of the circulant impedance matrix**) are given by:

$$\begin{bmatrix} \bar{Z} - \left(-j\frac{\sqrt{3}}{2}\left(\bar{K} - \bar{M}\right) - \bar{K} - \bar{M} + 2\bar{Z} \right) & \bar{M} & \bar{K} \\ \bar{K} & \bar{Z} - \left(-j\frac{\sqrt{3}}{2}\left(\bar{K} - \bar{M}\right) - \bar{K} - \bar{M} + 2\bar{Z} \right) & \bar{M} \\ \bar{M} & \bar{K} & \bar{Z} - \left(-j\frac{\sqrt{3}}{2}\left(\bar{K} - \bar{M}\right) - \bar{K} - \bar{M} + 2\bar{Z} \right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

The solution is given by:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \left(\cos\frac{2pm}{3} + j\sin\frac{2pm}{3}\right)_{m=0} \\ \left(\cos\frac{2pm}{3} + j\sin\frac{2pm}{3}\right)_{m=2} \\ \left(\cos\frac{2pm}{3} + j\sin\frac{2pm}{3}\right)_{m=1} \end{bmatrix}$$

Observation: the eigenvectors of a circulant matrix of rank n are composed of the n roots of unity given by the solution of the equation $z^n = 1$ or z Î □ and n Î □.

Transformation of a three-phase system with symmetrical components

The **eigenvectors** of the root λ_3 (**corresponding to the case of the circulant impedance matrix**) are given by:

$$\begin{bmatrix} \bar{Z}-\left(j\frac{\sqrt{3}}{2}\left(\bar{K}-\bar{M}\right)-\bar{K}-\bar{M}+2\bar{Z}\right) & \bar{M} & \bar{K} \\ \bar{K} & \bar{Z}-\left(j\frac{\sqrt{3}}{2}\left(\bar{K}-\bar{M}\right)-\bar{K}-\bar{M}+2\bar{Z}\right) & \bar{M} \\ \bar{M} & \bar{K} & \bar{Z}-\left(j\frac{\sqrt{3}}{2}\left(\bar{K}-\bar{M}\right)-\bar{K}-\bar{M}+2\bar{Z}\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

The solution is given by:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \left(\cos\frac{2pm}{3}+j\sin\frac{2pm}{3}\right)_{m=0} \\ \left(\cos\frac{2pm}{3}+j\sin\frac{2pm}{3}\right)_{m=1} \\ \left(\cos\frac{2pm}{3}+j\sin\frac{2pm}{3}\right)_{m=2} \end{bmatrix}$$

Observation: the eigenvectors of a circulant matrix of rank n are composed of the n roots of unity given by the solution of the equation $z^n = 1$ or z Î □ and n Î □.

Transformation of a three-phase system with symmetrical components

Through the definition of the complex number a Î □

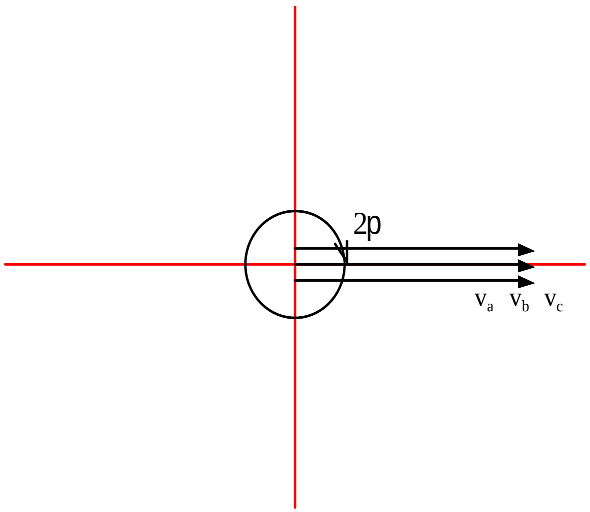
$$a = e^{j\frac{2}{3}p} = \cos\left(\frac{2}{3}p\right) + j\sin\left(\frac{2}{3}p\right)$$

we obtain the common eigenvectors for symmetrical and circulant matrices $\left[\bar{Z}_{abc}\right]$ **:**

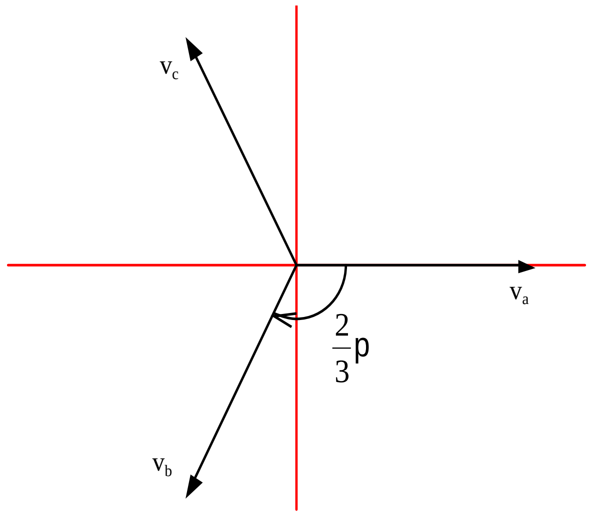
$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix}, \begin{bmatrix} 1 \\ a \\ a^2 \end{bmatrix}$$

The three vectors shown are called **sequences**

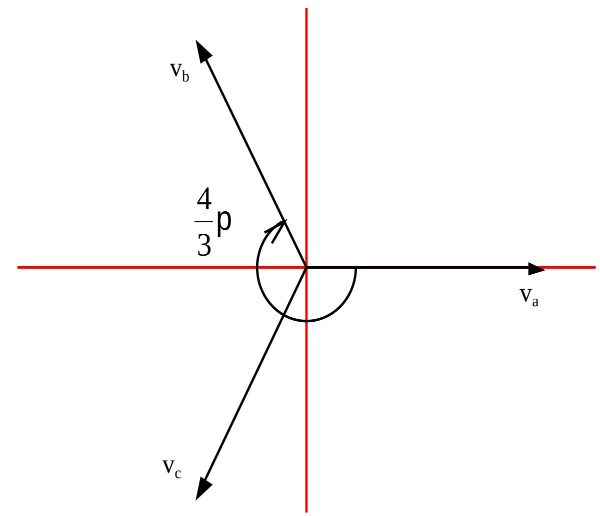
Transformation of a three-phase system with symmetrical components



Symmetrical **homopolar** system(**zero** sequence or **0**)



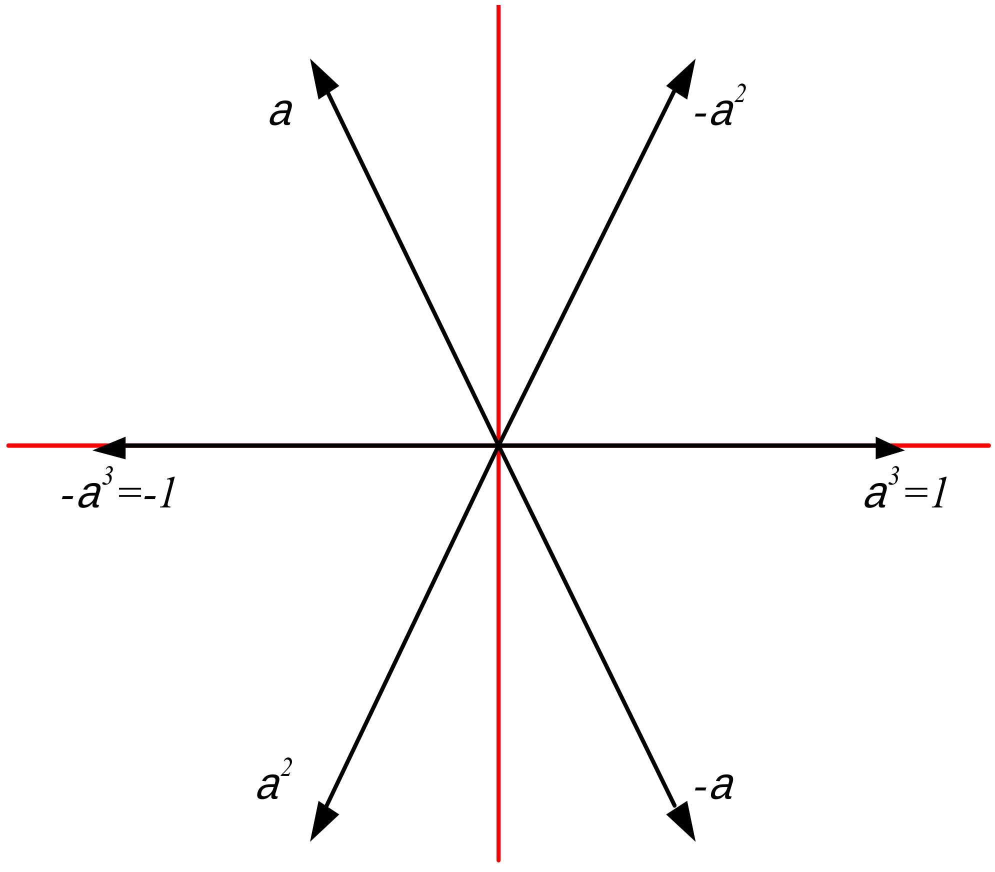
Direct symmetrical system (**positive** sequence or **1**)



Inverse symmetrical (**negative** sequence or **2**)

$$\left[S^0(\bar{V}_0)\right] = \bar{V}_0 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \left[S^1(\bar{V}_d)\right] = \bar{V}_d \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix} \qquad \left[S^2(\bar{V}_i)\right] = \bar{V}_i \begin{bmatrix} 1 \\ a \\ a^2 \end{bmatrix}$$

Transformation of a three-phase system with symmetrical components



Transformation of a three-phase system with symmetrical components

The matrix $[T_s]$ with eigenvectors in the columns, i.e. the matrix with sequences $\left[S^0(1)\right],\left[S^1(1)\right],\left[S^2(1)\right]$ in the columns has a **non-zero determinant**. This matrix represents a **change of basis of the vector space** $\mathbb{C}^3$

$$\left[T_s\right]=\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

In fact, the vectors $\left[S^0(1)\right],\left[S^1(1)\right],\left[S^2(1)\right]$ are linearly independent and form a **basis** for $\mathbb{C}^3$.

Transformation of a three-phase system with symmetrical components

Observation: **the previous sequence property means that any vector of $\mathbb{C}^3$, i.e. any set of three complex numbers representing a three-phase system, can be expressed as a linear combination of the elements of this base by the following equation:**

$$\begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \bar{V}_0 \\ \bar{V}_d \\ \bar{V}_i \end{bmatrix} = \left[T_s \right] \begin{bmatrix} \bar{V}_0 \\ \bar{V}_d \\ \bar{V}_i \end{bmatrix}$$

Transformation of a three-phase system with symmetrical components

Observation: therefore, for a **generic element of a three-phase system** $\left[\bar{V}_a, \bar{V}_b, \bar{V}_c\right]$, the **homopolar**, **direct** and **inverse components** $\left[\bar{V}_0, \bar{V}_d, \bar{V}_i\right]$ can be determined by:

$$\begin{bmatrix} \bar{V}_0 \\ \bar{V}_d \\ \bar{V}_i \end{bmatrix} = \left[T_s\right]^{-1} \begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix}$$

Transformation of a three-phase system with symmetrical components

Now we can write the equation in compact form

$$\begin{bmatrix} \bar{E}_a \\ \bar{E}_b \\ \bar{E}_c \end{bmatrix} = \begin{bmatrix} \bar{Z}_{aa} & \bar{Z}_{ab} & \bar{Z}_{ac} \\ \bar{Z}_{ba} & \bar{Z}_{bb} & \bar{Z}_{bc} \\ \bar{Z}_{ca} & \bar{Z}_{cb} & \bar{Z}_{cc} \end{bmatrix} \begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix}$$

$$\left[\bar{E}_{abc}\right] = \left[\bar{Z}_{abc}\right]\left[\bar{I}_{abc}\right]$$

and the triplet of **voltages and currents can be decomposed** by the matrix $[T_s]$, resulting in:

$$\left[T_s\right]\left[\bar{E}_{0di}\right] = \left[\bar{Z}_{abc}\right]\left[T_s\right]\left[\bar{I}_{0di}\right]$$

$$\left[\bar{E}_{0di}\right] = \left[T_s\right]^{-1}\left[\bar{Z}_{abc}\right]\left[T_s\right]\left[\bar{I}_{0di}\right]$$

Transformation of a three-phase system with symmetrical components

If we consider that $\left[T_s\right]^{-1}\left[\bar{Z}_{abc}\right]\left[T_s\right]$ gives rise to a diagonal matrix whose **diagonal terms are the eigenvalues** of $\left[\bar{Z}_{abc}\right]$, we obtain for the first type of matrix:

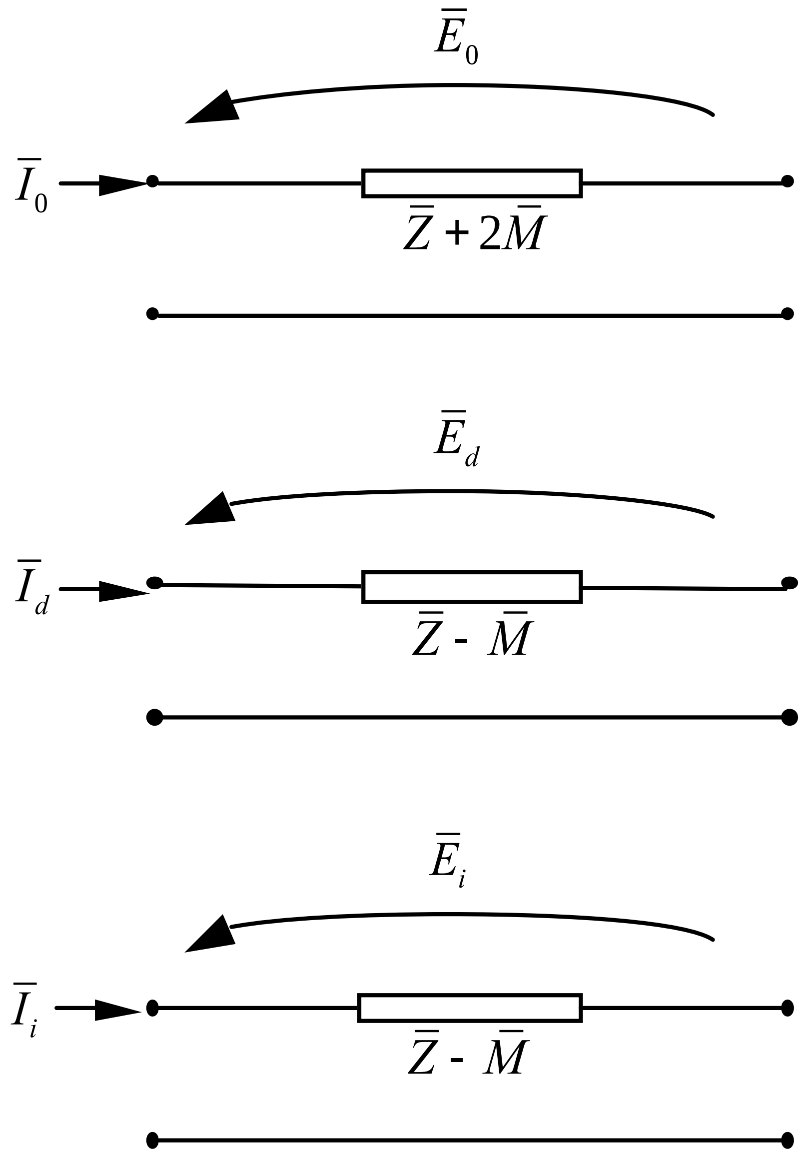
$$\begin{bmatrix} \bar{E}_0 \\ \bar{E}_d \\ \bar{E}_i \end{bmatrix} = \begin{bmatrix} \bar{Z}+2\bar{M} & 0 & 0 \\ 0 & \bar{Z}-\bar{M} & 0 \\ 0 & 0 & \bar{Z}-\bar{M} \end{bmatrix} \begin{bmatrix} \bar{I}_0 \\ \bar{I}_d \\ \bar{I}_i \end{bmatrix}$$

Therefore, **the generic network element supplied by a generic voltage system** (asymmetrical and unbalanced) and a **generic current system** (also asymmetrical and unbalanced), can be **decomposed into three three-phase circuits where they act only in the homopolar, direct and reverse sequence.**

Transformation of a three-phase system with symmetrical components

Each three-phase circuit can be represented by an equivalent single-phase circuit as shown in the figure.

The circuits in the figure **are not coupled → there is no electrical connection between the sequence circuits, so they can be solved separately.**



Outline

Transformation of a three-phase system with symmetrical components

Considerations for Grounding

Decoupling of an asymmetrical three-phase impedance system

Apparent power invariance property

Considerations for Grounding

The sequence voltages and currents of a three-phase system can be used to **make observations** about the **grounding** characteristics of the system.

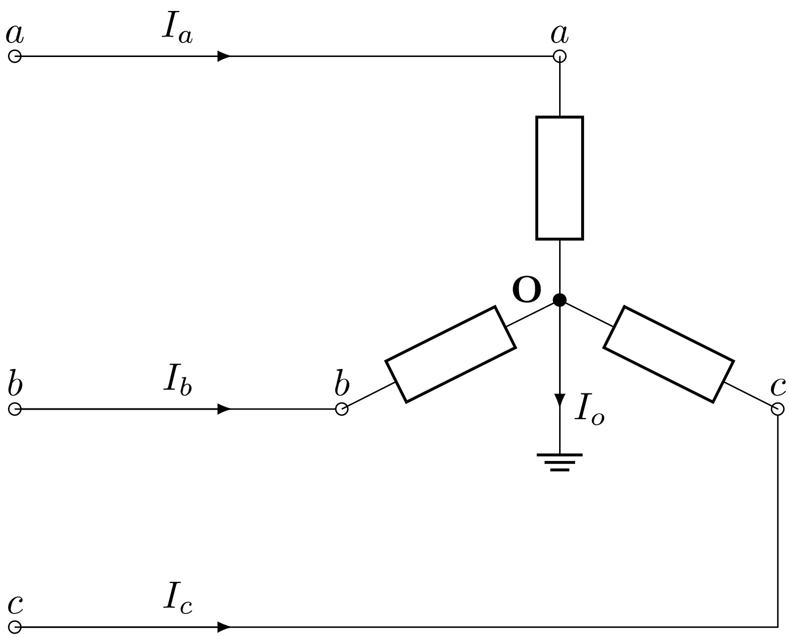
Definition: **grounded system** – a system in which **the point at the center of the phase-to-ground voltages is connected to earth via a low or zero impedance**.

If the impedance of the grounding system is zero (**ideal grounding**), the **homopolar (zero sequence) voltage is equal to zero and the homopolar current may be non-zero.**

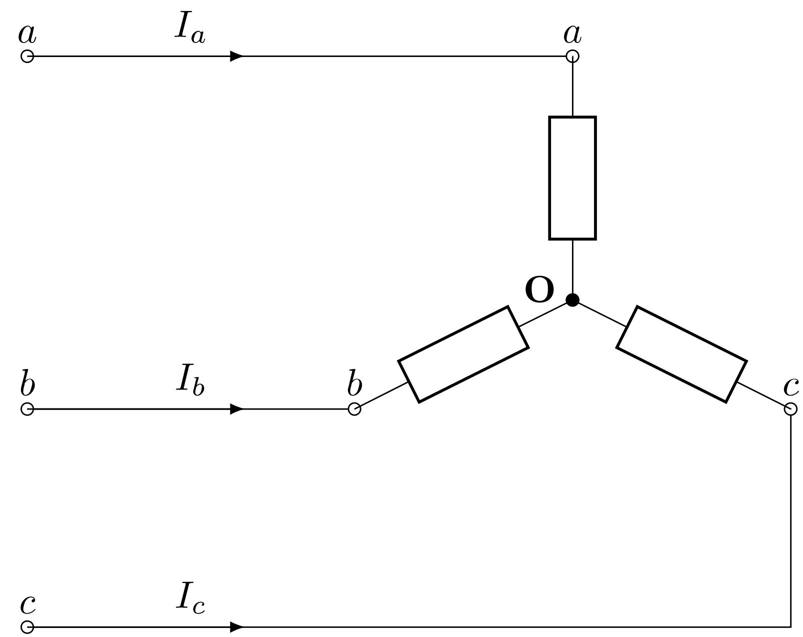
If the system is **ungrounded**, the **homopolar current is equal to zero** and the **homopolar voltage may be non-zero**.

Considerations for Grounding

If the impedance of the grounding system is zero (**ideal grounding**), the **homopolar (zero sequence) voltage is equal to zero and the homopolar current may be non-zero.**
If the system is **ungrounded**, the **homopolar current is equal to zero** and the **homopolar voltage may be non-zero.**



$$\bar{I}_a + \bar{I}_b + \bar{I}_c = \bar{I}_o$$
$$\bar{E}_a + \bar{E}_b + \bar{E}_c = \bar{E}_o = 0$$



$$\bar{I}_a + \bar{I}_b + \bar{I}_c = \bar{I}_o = 0$$
$$\bar{E}_a + \bar{E}_b + \bar{E}_c = \bar{E}_o$$

Outline

Transformation of a three-phase system with symmetrical components

Considerations for Grounding

Decoupling of an asymmetrical three-phase impedance system

Apparent power invariance property

Decoupling of an asymmetrical three-phase impedance system (asymmetrical line)

We examine the **case of a line whose impedance matrix does not have a circulant structure**. We study a simple case, where the line can be represented only by **three different longitudinal impedances:**

$$\bar{Z}_{aa} \; 1 \; \bar{Z}_{bb} \; 1 \; \bar{Z}_{cc}$$

$$\bar{Z}_{ab} = \bar{Z}_{ac} = \bar{Z}_{bc} = \bar{Z}_{ba} = \bar{Z}_{ca} = \bar{Z}_{cb} = 0$$

Donc

$$\left[\bar{Z}_{abc}\right] = \begin{bmatrix} \bar{Z}_{aa} & 0 & 0 \\ 0 & \bar{Z}_{bb} & 0 \\ 0 & 0 & \bar{Z}_{cc} \end{bmatrix}$$

Decoupling of an asymmetrical three-phase impedance system (asymmetrical line)

If the matrix $[T_s]$ is also the matrix of eigenvectors of $\left[\bar{Z}_{abc}\right]$ with the result $\left[T_s\right]^{-1}\left[\bar{Z}_{abc}\right]\left[T_s\right]$ we should get a diagonal matrix, and instead we get:

$$\left[T_s\right]^{-1}\left[\bar{Z}_{abc}\right]\left[T_s\right]=\frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & a & a^2\\ 1 & a^2 & a\end{bmatrix}\begin{bmatrix}\bar{Z}_{aa} & 0 & 0\\ 0 & \bar{Z}_{bb} & 0\\ 0 & 0 & \bar{Z}_{cc}\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\ 1 & a^2 & a\\ 1 & a & a^2\end{bmatrix}=$$

$$=\frac{1}{3}\begin{bmatrix}\bar{Z}_{aa}+\bar{Z}_{bb}+\bar{Z}_{cc} & \bar{Z}_{aa}+a^2\bar{Z}_{bb}+a\bar{Z}_{cc} & \bar{Z}_{aa}+a\bar{Z}_{bb}+a\bar{Z}_{cc}\\ \bar{Z}_{aa}+a\bar{Z}_{bb}+a\bar{Z}_{cc} & \bar{Z}_{aa}+\bar{Z}_{bb}+\bar{Z}_{cc} & \bar{Z}_{aa}+a^2\bar{Z}_{bb}+a\bar{Z}_{cc}\\ \bar{Z}_{aa}+a^2\bar{Z}_{bb}+a\bar{Z}_{cc} & \bar{Z}_{aa}+a\bar{Z}_{bb}+a\bar{Z}_{cc} & \bar{Z}_{aa}+\bar{Z}_{bb}+\bar{Z}_{cc}\end{bmatrix}$$

Outline

Transformation of a three-phase system with symmetrical components

Considerations for Grounding

Decoupling of an asymmetrical three-phase impedance system

Apparent power invariance property

Apparent power invariance property

When using the sequence decomposition of a three-phase electrical system, the **invariance of the complex power** must be taken into account, given the equivalence demonstrated below between the original three-phase circuit and the sequence circuits.
This property is easily demonstrated by considering the expression of the apparent power of a three-phase circuit fed by three simple voltages $\bar{E}_a, \bar{E}_b, \bar{E}_c$, which absorb three line currents $\bar{I}_a, \bar{I}_b, \bar{I}_c$

$$\bar{S} = \bar{E}_a \bar{I}_a^* + \bar{E}_b \bar{I}_b^* + \bar{E}_c \bar{I}_c^*$$

This equation can be written as the scalar product of the following vectors:

$$\bar{S} = \left[\bar{E}_{abc}\right]^t \cdot \left[\bar{I}_{abc}^*\right] = \left[\left[T_s\right]\left[\bar{E}_{0di}\right]\right]^t \cdot \left[T_s^*\right]\left[\bar{I}_{0di}^*\right] = \left[\bar{E}_{0di}\right]^t \left[T_s\right]^t \cdot \left[T_s^*\right]\left[\bar{I}_{0di}^*\right]$$

Apparent power invariance property

The product $[T_s]^t \cdot [T_s^*]$ is equal to

$$[T_s]^t [T_s^*] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} = 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore:

$$\bar{S} = [\bar{E}_{0di}]^t [T_s]^t \cdot [T_s^*][\bar{I}^*_{0di}] = 3\bar{E}_0\bar{I}_0^* + 3\bar{E}_d\bar{I}_d^* + 3\bar{E}_i\bar{I}_i^*$$

Since the three sequence circuits $0,d,i,$ are symmetrical, their apparent powers can be written as:

$$\bar{S}_0 = 3\bar{V}_0\bar{I}_0^* \qquad \bar{S}_d = 3\bar{V}_d\bar{I}_d^* \qquad \bar{S}_i = 3\bar{V}_i\bar{I}_i^*$$

Therefore, we have **energetic equivalence of the three-phase circuit and its composing sequence circuits.**